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BASIC REFERENCE FRAMES AND TRANSFORMATIONS FOR AIR-TO-AIR GUNNE--ETC(U)

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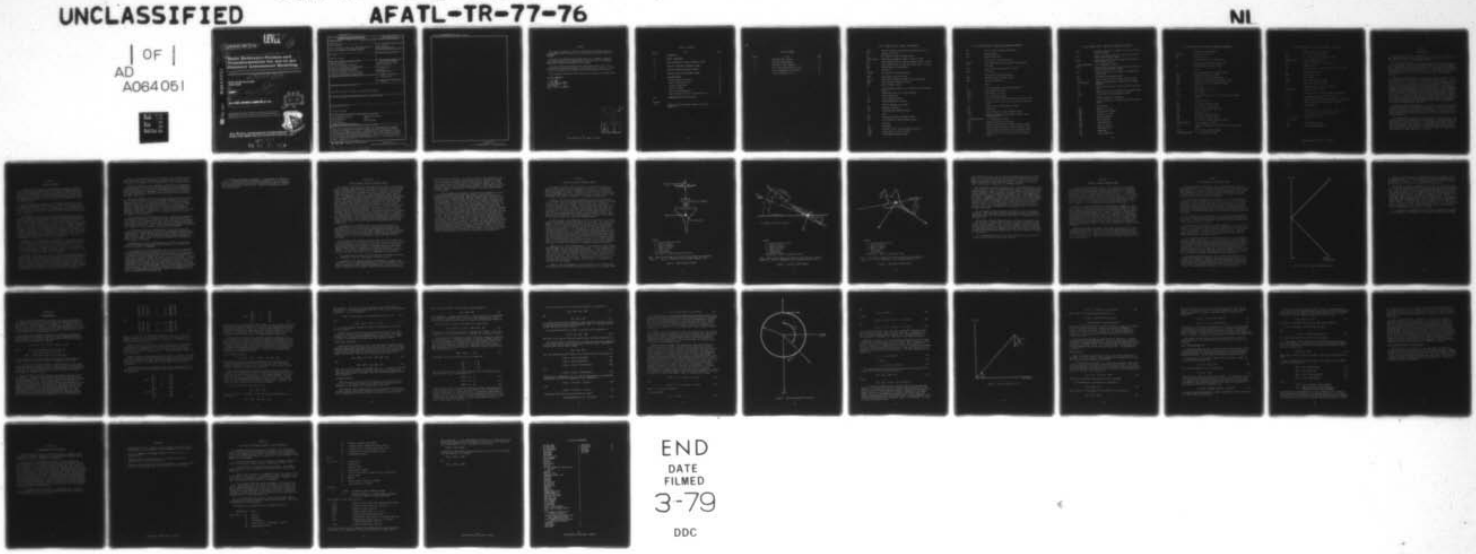

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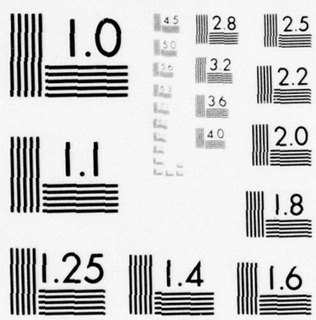

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Basic Reference Frames and Transformations for Air-to-Air Gunnery Assessment Modeling

(10) Harold E. /Smith

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This report defines basic reference frames and transformations which are offered as standards in Air-to-Air Gunnery Assessment Modeling (ATAGAM). Adoption would make this report a documentation of these elements of about 12 known USAF ATAGAM models while minimizing their bookkeeping requirements and increasing their general comprehension. The basics are applicable to other conventional weapon delivery methodologies i.e., for missiles, rockets, bombs, cannisters, etc.

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PREFACE

This report documents an effort to standardize reference frames and transformations that are used in Air-to-Air Gunnery Assessment Modeling (ATAGAM).

This work was conducted during April-May 1977 in support of Project 2543, Weapons Effectiveness Methodology, and by the Air Force Armament Laboratory, Armament Development and Test Center.

This report has been reviewed by the Information Officer (OI) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nations.

This technical report has been reviewed and is approved for publication.

FOR THE COMMANDER

J R Murray

J. R. MURRAY
Chief, Analysis Division

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LIST OF ABBREVIATIONS, SYMBOLS AND ACRONYMS

A/B/C	General notation for angular orientation about x/y/z
ADL	Armament datum line
AG	Gun azimuth in body reference frame
alpha, AOA, α	Wind angle component in body's vertical plane
AO	Angle between line of sight and target velocity vector
AOT	Angle between line of sight and target body - x vector
ARL	Applied Research Laboratories
ATA	Air-to-air
ATAGAM	ATA gunnery assessment modeling
BBRF	Bullet body reference frame
beta, β	Wind angle component in body's horizontal plane
BNRF	Bullet notation reference frame
BRF	Body reference frame
C	Prefix denoting element of a rotational transformation matrix
C	Shortening prefix for cosine function
cg	Center of gravity
CPI	Cross product inertia
CSS	Closed-form Siacci solution
D	General notation of an angle
DOF	Degrees of freedom
DRF	Disturbed-axes reference frame
E	East
EBRF	Exterior ballistics reference frame
EG	Gun elevation in body reference frame
f	Function
FB	Fixed-body
FCN1	Function 1 used in pitch aspect definition
FIRF	Fixed inertial reference frame
FSRF	Fixed-sight reference frame

LIST OF ABBREVIATIONS, SYMBOLS AND ACRONYMS (CONTINUED)

GVD	Gravity vector directed (direction)
HASP	Heading aspect
HOT	Higher order term
L	Subscript for launch direction
LCGCRF	Lead computing gyro case/caged reference frame
LH	Left hand
LOS	Line of sight (unit vector direction)
LOSR	LOS range
M	Subscript for muzzle direction
MIRF	Moving inertial reference frame
MPM	Modified point-mass
N	North
P	Siacci coordinate along launch direction
PASP	Pitch aspect of target
p/q/r	Rate about x/y/z axis
P/Q/R	p/q/r, alternate symbol
PAI	Attacker position in fixed inertial reference frame
PLI	Sight-to-target vector in fixed inertial reference frame
PLRF	Pipper line of sight reference frame
PM	Point mass
PMB	Muzzle position in body reference frame
PNLB	Target-to-sight position vector in target body reference frame
PNLBX/PNLBY/PNLBZ	x/y/z components of PNLB
PNLI	Target-to-sight position vector
PQB	General-object position in body reference frame
PQE	General-object position in earth reference frame
PQW	General-object position in wind reference frame
PPI	Projectile position in fixed inertial reference frame

PPLI	Projectile position at launch in the fixed inertial reference frame
PPS	Relative projectile position from sight origin in fixed sight reference frame
PPSB	Relative projectile position from sight origin in body reference frame
PPSBX/PPSBY/PPSBZ	x/y/z components of PPSB
PPSI	Relative projectile position from sight origin in fixed inertial reference frame
PPT	Projectile position in trajectory initiation reference frame
PRF	Principal axes reference frame
PSB	Sight-origin position in body reference frame
psi/theta/phi	Heading (bearing)/pitch (elevation)/roll (bank) attitude
PSI	Sight-origin position in fixed inertial reference frame
PTI	Target position in fixed inertial reference frame
P/V/A	Position/velocity/acceleration vector
Q	General notation of an object
Q	Siacci coordinate in the direction of the gravity vector
R	Range vector
RAB	General B to A RT
RASP	Roll aspect of target
RBA	General A to B RT
RBE	Earth-to-body RT
RBG	Gun-to-body RT
RBW	Wind-to-body RT
REB	Body-to-earth RT
RF	Reference frame
RH	Right hand
RSB	Body-to-sight RT

LIST OF ABBREVIATIONS, SYMBOLS AND ACRONYMS (CONTINUED)

RT	Rotational transformation
RX/RY/RZ	RT about x/y/z axis
RWB	Body-to-wind RT
R1	RT about general axis, denoted by 1
S	Shortening prefix for sine function
SRF	Stability-axes reference frame
t	Time
T	Shortening prefix for tangent function
TBRF	Target body reference frame
TFRF	Trajectory flight reference frame
TIRF	Trajectory initialization reference frame
TLRF	Target line of sight reference frame
U	University
USAF	United States Air Force
UV	Unit vector
UVTXI	UV along target x-axis in fixed inertial reference frame
VA	Attacker velocity
VAB	VA in body reference frame
VAW	VA in wind reference frame
VG-DG	Vertical gyro-directional gyro
VM	Muzzle velocity
VMB	VM in body reference frame
VMG	VM in gun reference frame
VMI	VM in fixed inertial reference frame
VMIX/VMIY/VMIZ	x/y/z components of VMI
VMT	Tangential component of VM due to aircraft rotation rate
VMTB	VMT in body reference frame
VMTBX/VMTBY/VMTBZ	x/y/z components of VMTB

LIST OF ABBREVIATIONS, SYMBOLS AND ACRONYMS (CONCLUDED)

VL	Launch velocity of projectile
VLB	VL in body reference frame
VLI	VL in fixed inertial reference frame
VLIX/VLIY/VLIZ	x/y/z components of VLI
VNB	Negative of bullet velocity vector
VT	Target velocity
VTI	Target velocity in fixed inertial reference frame
WB	Weights and balances
WBRF	WB reference frame
WC	Wing chord
WCRF	Wing chord reference frame
WRF	Wind-axes reference frame
XZ	Aircraft vertical plane of symmetry
XI/ETA/ZETA	ARL RH reference frame
y	Subscript for initial wind angle on projectile
$\xi/\eta/\zeta$	XI/ETA/ZETA in Greek letter notation
Σ	Angle between bullet nose and velocity directions
Φ	ARL notation for bullet precession angle
$\psi/\theta/\phi$	psi/theta/phi
π	pi
ω	omega, Greek letter notation for angular rate
$\tilde{\omega}$	Tilde operation, changes $\omega \otimes R$ to $\tilde{\omega}R$
$\dot{(\ )}$, ( ) dot	Time derivative of ( )
( ) - sub A	$(\ )_A$
$\odot$	Dot product operator
$\otimes$	Cross product operator

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SECTION I

INTRODUCTION

This report is offered for standardizations in air-to-air gunnery assessment modeling (ATAGAM). While directed toward this charter it is also applicable to all forms of aerial weapon delivery.

The treatise evolved from an original intent to clarify certain aspects of specific ATAGAM. Clarification required definition of the basic framework. Suggestions from associates expanded it, so that only a few additions were necessary to achieve the present form. It is the apparent solution to the quandary of documenting each and every model which uses essentially the same transformations as about 12 other models within the USAF ATAGAM community.

There are 24 different ways of indexing an orthogonal reference frame (RF) along the basic directions of 3-dimensional Euclidean space; i.e., the six basic directions for one axis times the four cardinal points of rotation for another axis. Directions of two axes of an orthogonal RF defines the third. For each indexing, there are six different ways of ordering the rotations of heading/bearing/psi, elevation/pitch/theta, and roll/bank/phi (throughout, the Greek notations will be generally used to denote specified rotations after associations with common names have been made). There are also six different transformations for the three primary reference frames of flight; i.e., inertial, body, and wind. When considerations are made for a variety of secondary reference frames and multiple bodies of interest, it is readily apparent that bookkeeping is a major concern. Standards of indexing, rotational ordering, and inter-reference-frame transformations that minimize bookkeeping are preferred.

Conciseness has been subordinated to an attempt at thorough explanations to beginning Scientific and Engineering personnel. Knowledge of right hand and left hand RFs including indexing and sensing is assumed. The great majority of mathematical operations in the development is defined by matrix/linear algebra with occasional inclusions of vector dot-product and cross-product operations.

To simplify report preparations, context is used to define vectors and matrices, instead of the bold-faced type convention. Vector components are identifiable by the addition of X, Y, or Z to the vector symbol. Dot and cross product operations are denoted by $\odot$ and $\otimes$, respectively, between the vector symbols. Also, the sub ( ) notation is used to save lines that would be lost by using subscripts. No distinctions have been made in the use of upper and lower case use of x, y, and z.

SECTION II

GENERAL BACKGROUND

In analytical studies and associated modeling, confusion sometimes arises concerning the definition of the direction of linear/angular quantities or of transformations, or both, that influence gunnery solutions. In some cases, the modeler may take equations from a text or report that may not be either correct or directly applicable to the problem at hand. The following discussions are offered to provide both insight and a basis for standardization.

For development, the references are considered *excellent treatises*, providing good fundamentals. Unfortunately, definitions and purposes may be somewhat vague to those with lesser experience. Also, certain authors may use unique definitions which cause undue complications when merged with overall modeling. This effort is directed for the consistency which avoids these complications.

The fundamentals should be carefully applied, since errors are ever present. Reference 1 uses numerous restricted/limited definitions that are not correct for general accounting. Examples are included in Section IV. The pz of the v-sub x equation (5.1, 6) of Reference 2 should be qz. Also, the 3 in *the z-sub V, z-sub 3 axis of Figure 4.9* in Reference 2 should be 2. Further, in the discussion on stability axes (4.2.7), it may be interpreted that the X-axis must lie in the aircraft (xz) plane of symmetry, while Reference 1 requires it to be parallel to the wind vector. Obviously, an aircraft can be stable with a steady-state sideslip/beta angle. The author's restricted definition can be interpreted as an omitted step to direct correlations with vertical (xz) plane stability derivatives. The sideslip transformations in References 3 and 4 are erroneous.

References 1 and 2 define beta as measured from a fixed-body x axis to the xy-plane wind. To be consistent for the RF, it should be shown wind-to-body, similar to the angle-of-attack (AOA)/alpha definition. Then it would be obvious that beta is oppositely-sensed to psi. Of course, all flight instrumentation including wind vanes is affixed to the airframe; and wind angle components can be consistently shown to the airframe. This discussion illustrates an example of mixed sensing.

References 1 and 2 use P, Q, and R for angular rates about the local RF (body or wind). While Reference 1 uses p, q, and r as small-perturbation (delta) change in steady-state values, Reference 2 uses p, q, and r as absolute rates that are corrected for both earth rotation and navigational changes. The most commonly used practice over the years of using p, q, and r for body/wind local rates is proffered as standard; P, Q, and R being used alternately (and obviously for all-capitals Fortran coding). Primes or other distinctions should be used for special case considerations.

Finally, the RF definitions of all references are clear and concise for author intent; but there are so many which address special cases that do not relate to the general order of accounting for aerial gunnery problems.

Reference 1 presents a most comprehensive description of aerodynamic forces and moments up through the disturbed-stability RF while Reference 2 describes origins of RFs and inter-RF developments with the modern matrix/ linear algebra approach. References 3 and 4 are masterworks of overview in exterior ballistics developments, though neglecting relative geometry and other simple codes.

Before attempting to uncover the confusing ramifications of the various RFs, it appears warranted to discuss the biggest problem that generally submerges the subleties of some interrelations. The major cause of confusion and mistakes is the mixing of RH and LH RFs, or indices, followed closely by using different ordering of rotations from the standard psi-theta-phi. While any consistent system is correct, the bookkeeping is generally overwhelming. No generality is lost by defining each RF as RH and indexing it back to its primary RF. This preferred method will minimize the number of transformations.

The proliferation of mixed/erroneous RFs has generally been caused by the desires of the modeler to conform with older conventions such as the U.S. Army Artillery convention of X-East, Y-North, and Z-Up and the gunner's plane (impact plane) where high and right misses were defined as positive Y and X misses, respectively. These conform to the lower mathematics graphical plot convention. Mixed sensings or conversions were necessary.

In engineering practices, X-North/forward, Y-East/right, and Z-GVD (gravity vector directed)/down has the obvious attribute of agreeing with conventional psi, theta, and phi definitions and alignment with the external force, gravity. North, East and GVD are associated with accounting-RF indices, while forward, right, and down are associated with rotatable RFs of the objects of interest.

Interpretations in the old conventions can be readily accomplished after standard RH-transformations if deemed necessary. When used, clear notation of intent is recommended.

It is noted that a projectile/bullet that flies through the atmosphere is an aerodynamic body that is affected quite similarly to aircraft with the exception that aircraft have variable controls/surfaces and power-plants to maneuver and sustain energy/flight. Thus, RFs for accounting of the bullet states should be similar but simpler to those for aircraft. Invariant configuration, ballistic flight, and no secondary/internal RFs greatly simplify the accounting system. However, there may be as many as 30 or more accountable (i.e., between firer and target fly-by) projectiles, requiring stored state arrays in revolving tables that drop those beyond fly-by and add newly-fired ones. Generally, nominal trajectories are computed for initiations at the beginning of each iteration interval and interpolations are made in accordance with the firing rate.

In concluding the general background, it is noted that all modeling is relative. Though some factors are known or suspected to vary in the real world, models neglecting these factors are chosen where they will hopefully have negligible impact on the results. Objective interpretations should always be qualified.

SECTION III

FIXED AND MOVING INERTIAL REFERENCE FRAMES

The absolute inertial RF has been defined as the center of the universe which is academic and undefinable. The galactic-inertial RF is of the same stature, but manageable in a restricted sense. A geocentric (earth-center origin) inertial RF can be defined for multi-body orbital flight with the Z directed toward the sun and the xz plane defining the earth's orbital plane. For atmospheric flight, some simplified models consider the earth as a fixed spheroid, correcting the solution to varying degrees of sophistication for earth rotation and/or gravity vector anomalies afterward. Some sophisticated models consider rotation, oblateness, and curve-fitted magnetic anomalies; but obviously avoid the mechanics of rapidly varying anomalies where platforms are described to go wild. The rotating geocentric RF has its Z-axis directed through the South spin pole and its X-axis directed through the Greenwich Meridian. The earth-surface RF is directly applicable to the navigational problem. Initialized at home-base, solutions determine aircraft state relative to the earth and to points of interest; i.e., destinations. Sophisticated internal mechanizations are included to direct/tune X-North, Y-East, and Z-GVD and to compensate for earth rotation. These are important to space flight or long-term navigation, but are obviously overly complex to the few seconds or less characteristics of the gunnery pass.

An important one for ATAGAM is the local attitude-indexing RF; i.e, the true verticality and directionality sensed by either the Vertical-Directional Gyro (VG-DG) Set or the Inertial Reference Platform. Note, these are the true indices, not the sensor outputs nor the attitude outputs/relations of the aircraft/weapon body RFs to these indices, discussed in the next section. This is the moving-base inertial RF (MIRF) of the references.

The accuracy of the VG-DG Set with drift rates of up to 20 deg/hr for accelerated flight can significantly affect some long-range gunnery problems, but most contemporary ones include local-gravity erection or caging techniques which obviates this concern for reasonable flight profiles, while absolute directionality is generally not a significant factor in aerial gunnery. Also, the RF is usually either physically located very close to the aircraft center of gravity (cg) or is compensated for the offset.

It is noted that the origin of all attitude RFs for ATAGAM is the body cg. Variability of the cg is discussed in the next section.

To account for the relative geometry of attacker/firer, target, and projectiles, it is a common modeling practice to freeze the MIRF in inertial space at significant events during the gunnery pass. The primary one is used to account for firer, targets, and bullet attitudes and positions and

therefore relative geometry. Its origin is fixed at the initiation of the pass at the firer's cg at the altitude/negative-z of interest; the target is initialized with respect to the firer in this RF. Obviously, the flat earth approximation is sufficient for aerial gunnery ranges. This is the fixed inertial RF (FIRF) used for basic accounting. Note, MIRF and FIRF are parallel. Both are called earth in some models, overview being necessary for discernment.

In the more sophisticated models, projectile fly-out modeling is incorporated and compared with stored target states to define fly-by; i.e., intersection in the impact plane. The most commonly used fly-out/exterior-ballistics code is the point-mass (PM) model which assumes zero lift; i.e., no yaw of repose. Relative errors to the full six degree of freedom model are approximately in the few mils or less category for design gunnery; i.e., 1000 to 1500 feet line of sight (LOS) range and wind angles up to about 10 degrees. Because of the simplification of the PM model, a single plane coordinate system is sufficient. Some PM models operate open loop, iterating projectile travel in the X-forward and Z-down from drag (coefficient) versus mach tables and flight equations, while others use closed form equations such as the P-Q Siacci RF where P is forward along the line of departure and Q is down. Of course, the simplest model is the straight line with average velocity over the ranges of interest. Average velocity approximations are available for both inertial and relative ranges. Gravity drop of the no-lift body can also be easily calculated. Considering the gross approximations of the PM model and inclusion of aiming statistics that submerge exact details, there are branches of the aerial gunnery problem where simplified ballistics should be given consideration. Originated/frozen at muzzle-exit or time-of-fire, these are the exterior ballistics RFs (EBRFs). The single-plane EBRFs require transformations to the FIRF for impact-plane determinations.

SECTION IV

AIRCRAFT FIXED-BODY REFERENCE FRAMES

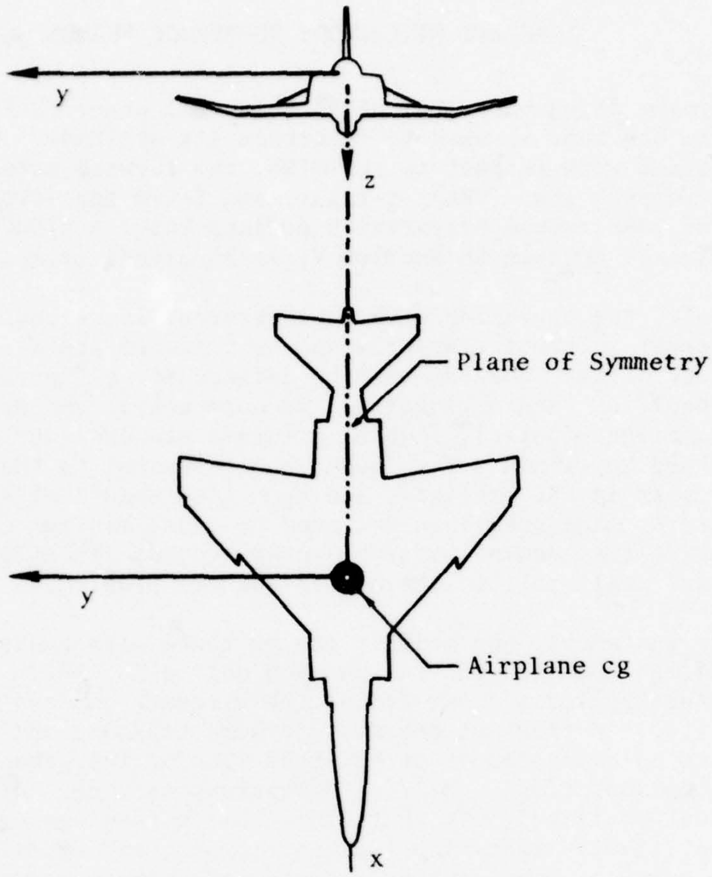
The primary fixed-body (FB) RF to which all other FBRFs should be related is the one that is used to reference its attitude. CG-originated and initialized with respect to the MIRF, the forward extension of the fuselage reference line (FRL), Y-right, and Z-GVD for level xy plane define this body RF (BRF) whose orientation defines Euler attitudes. The Y-right, a principal axis defined in Section V, is invariant in most FBRFs.

Obviously, the cg varies with fuel/armament loads and configuration; i.e., different internal gear/crew and/or variable states such as tilt/swing surfaces, etc. Thus, the FRL must be defined for either a nominal or critical condition such as landing. Because weight and balance (WB) is so critical to flight control, fighter aircraft are designed such that the cg is constrained to within a few inches about nominal in the fore and aft direction, less in the vertical, and even less than that laterally; i.e., fuel and stores management are designed to cause minimum cg shift. Thus, while cg shift from nominal must be acknowledged, its effect on the BRF can be considered negligible to the aerial gunnery problem.

In some instances, the modeler may be faced with converting cg and FRL from the design/WBRF (weight and balance RF) to the BRF. The origin of the WBRF is generally located outside of the aircraft to keep most dimensions positive; i.e., in front of the most forward tip/nose and either up-down located with an extension of an FRL-type line or the ground plane for clearance considerations. X, Y, Z dimensions in inches are given in fuselage stations positively aft, buttline (inside fuselage/empennage) or wing stations positively right-side looking forward, and waterlines positively up. While buttline wing station is measured from the symmetrical/xz plane, both origin and direction are arbitrary for the x/z axis, though an orthogonal RF. Most aircraft do not touch down or rest level on their landing gear. Thus, specifically when this RF is defined to show ground clearances, the slope of the FRL with respect to a reference waterline is required.

Figures 1 and 2, taken from Reference 1, are presented to show the Wing Chord RF (WCRF) and associated angle definitions. It is an important fixed body RF (FBRF), because lift occurs along negative z. (WC, called so because most lift is derived from the wings, is defined by the total lifting area.) Note that the pitch attitude of the WC is not the primary pitch attitude of the BRF. This frame is identical to the BRF except for a small rotation about the identical y-axis. Thrust and Drag RFs are obviously similarly small rotations of the BRF. Usually, drag is accounted for in the stability axes system.

Figure 3, also from Reference 1, but corrected for the aforementioned beta definition, shows another restricted definition; the associated angles

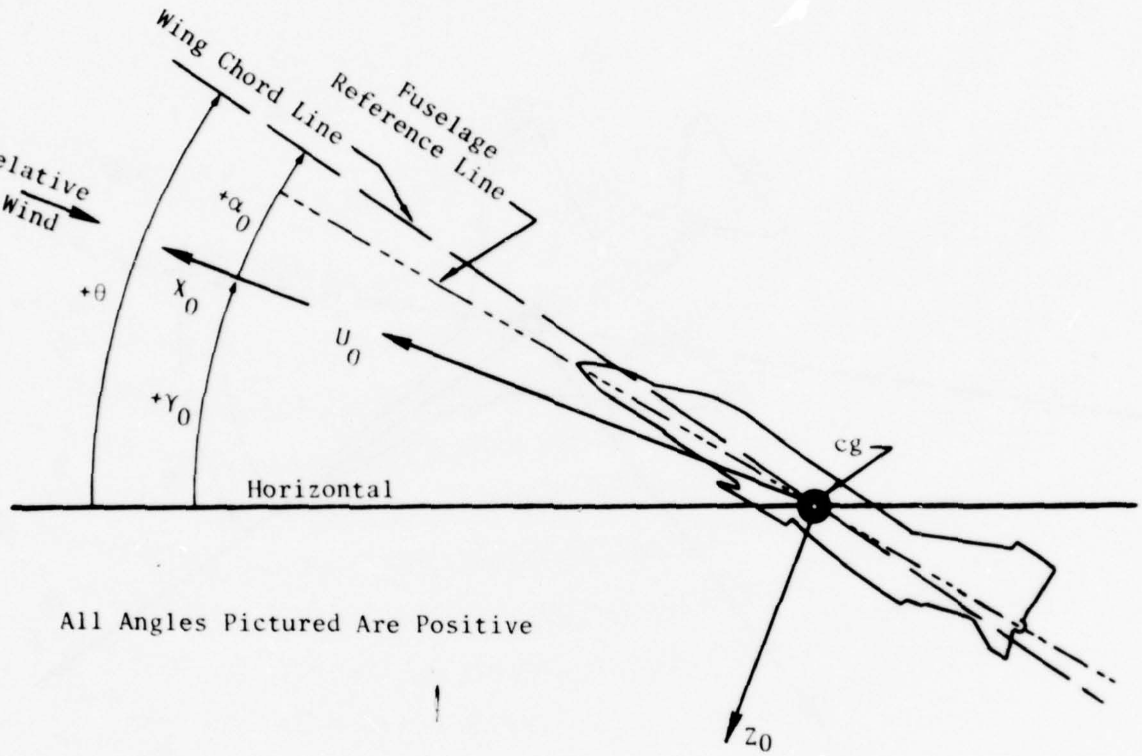


LEGEND:

U - Aircraft Velocity Vector
α - Angle of Attack
β - Angle of Sideslip
γ - Angle of Climb
θ - Pitch Angle
ψ - Heading Angle
Subscript 0 - Denotes steady-state value

NOTE: Vertical Plane Angles are shown for wing chord line reference (Figure 2). Pitch is referenced to the fuselage reference line.

Figure 1. Body Reference System



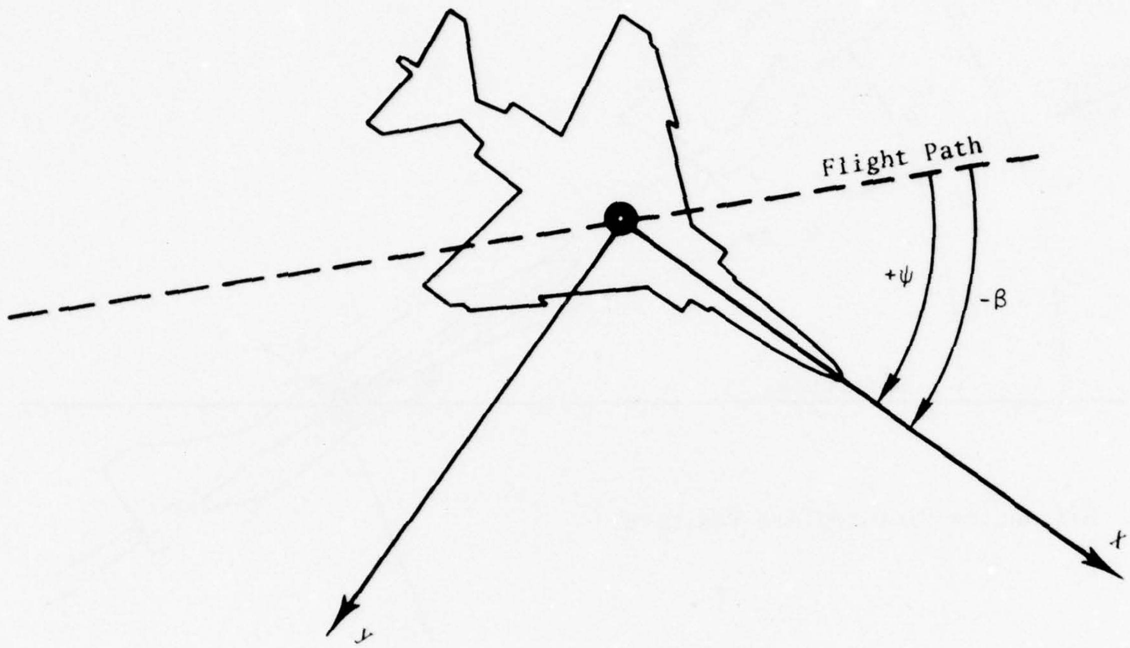
All Angles Pictured Are Positive

LEGEND:

U - Aircraft Velocity Vector
α - Angle of Attack
β - Angle of Sideslip
γ - Angle of Climb
θ - Pitch Angle
ψ - Heading Angle
Subscript 0 - Denotes steady-state value

NOTE: Vertical Plane Angles are shown for wing chord line reference (Figure 2). Pitch is referenced to the fuselage reference line.

Figure 2. Vertical Plane Geometry



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ψ - Heading Angle
Subscript 0 - Denotes steady-state value

NOTE: Vertical Plane Angles are shown for wing chord line reference (Figure 2). Pitch is referenced to the fuselage reference line.

Figure 3. Horizontal Plane Geometry

in the horizontal plane. Psi is shown to be sensed relative to the flight path. This is only true when the flight path is North-directed; and, in general, the flight path may have any orientation. Thus, the psi in the figure should be more aptly labeled delta-psi. Note that z is into the page. Accordingly, β is positive for left-hand rotation.

The fixed-sight RF (FSRF) originates on the sight combining glass. While X may be parallel to BRF X in certain vehicles, it may be along the primary armament datum line (ADL) in others or multiple FSRFs may be defined for the various armaments. The ADL specifies the nominal line of departure at the design point altitude and airspeed for missile, rocket, and/or bomb weapon deliveries. In gunnery, it is the harmonization point; i.e., where the gun will shoot at harmonization range, usually 2250 feet in front of the parked/jacked-up aircraft. The gunnery ADL is usually identifiable by either an etched gun-cross reticle or the caging (Zero Sight Line) of the movable-reticle. Rarely is the zero-sight and/or ADL displaced from the body symmetrical plane. Thus, the FSRF/ADL RFs are generally small or no y-axis rotations of the BRF.

The lead computing gyro case/caged RF (LCGCRF) defines zero gunsight deflection. While some older sights rotated the case about y for rolling sensitivity, the x-axis of some of the newer LCGCs is aligned parallel to the gunnery ADL.

The gun RF is similarly defined except it is rotated about both y and z axes of the BRF in harmonization to the ADL and also to correct for both gravity drop and parallax. Note, no roll is defined, irrespective of gun receiver block orientation. One exception to parallax-gravity correction is noted where the gun fires parallel to the BRF x-axis and all corrections are made within the gunsight mechanization. The ADL is parallel to the FRL in this instance.

It is recommended that all of the above definitions be in the dash 34 Technical Orders for USAF aircraft of interest.

SECTION V

AIRCRAFT VARIABLE REFERENCE FRAMES

The principal-axes RF (PRF) defines a set of axes for which guidance control inputs most nearly cause couplets, pure rotations of the airframe. This simplifies flight equations by uncoupling the axes; i.e., rotation about one axis does not affect rotations in the other two. In aerodynamic terminology, the cross-product inertias (CPIs) are zeros. It is in this RF that the primary aerodynamic coefficients/stability-derivatives are defined. Of course, the PRF varies with flight condition and configuration. Most modeling ignores this RF, because flight is also defined for the stability axes RF (SRF); i.e., the coefficients being derived for the SRF.

The SRF is rotated from the body by the wind angle components, alpha (AOA) and beta. The SRF is identical to the wind-axes RF (WRF) for steady-state flight conditions and is the basis of development of the small-perturbation-theory-derived disturbed-axis RF (DRF). The DRF is identifiable as the time varying WRF/SRF output of the model; transformations being made with respect to the instantaneous values of the WRF/SRF. It is noted that there are beta (negative psi) and alpha (theta) rotations between the BRF and WRF/SRF but no roll. Note, CPIs are non-zeros, but most models ignore them. Of course, good flight/stability control augmentation systems are designed to compensate for CPIs. Accordingly, omissions are tolerable within limitations.

Similar to the FSRF, the dynamic sight pipper and target line of sight (LOS) RF (PLRF and TLRF) originate on the sight combining glass and like the WRF/SRF have no roll. They are expressly defined by the sight elevation (pitch) and azimuth (psi) deflections of their respective LOS with respect to the zero sight line. Pipper and target-centered impact plane definitions stem from these RFs.

SECTION VI

EXTERIOR BALLISTICS REFERENCE FRAMES

The Applied Research Laboratories (ARL) of the University of Texas authored References 3 and 4, using unique indexing of the RFs which requires mixed sensings. Section II indicated that this causes unnecessarily complicated bookkeeping. Also, the same RF is denoted differently in the two references and in various sections of Reference 3. The most distinctive notations were selected for presentation herein.

Figure 4 shows the differences in indexing of the basic and ARL RFs. The ARL inertial RF, denoted XI, ETA, ZETA (ξ, η, ζ), is rolled 270 degrees with respect to the basic RF, apparently done for the sake of calling up positively, initialized with ETA directed opposite to the gravity vector and XI directed along the bullet launch heading. Heading/elevation about the ARL Y/Z axes is oppositely/directly sensed to that of the basic RF. In the figure, N/E/FWD/RT/DWN have been used to abbreviate North/East/forward/right/down.

It is evident that the ARL author is consistent with indexing the RFs. The reader is cautioned that in following discussions, the equivalent ARL RF has the 270-degree roll difference in indexing from the basic which will not be explicity noted for each RF.

The trajectory initialization RF (TIRF), denoted I', J', K', by ARL is defined as the translation (PPLI, projectile launch position vector in the FIRF) of the gun muzzle at fire and launch heading (ψ-sub L) and elevation (θ-sub L) rotations to initial bullet direction in the FIRF. The TIRF establishes the basis for which a selection from the various ballistics models can be used to model the trajectory.

It is noted that TIRF X-FWD is identically the P axis of the Modified Point Mass (MPM) P-Q RF which is, of course, the RF for which ARL develops both the iterative MPM and Closed-form Siacci Solutions (CSS); the MPM being the apparent USAF ATAGAM standard although ARL suggests that the CSS is the ideal. Q is GVD from the tip of the P vector for clarity in visualizing bullet location, but the vector analyst can define Q from the TIRF origin with no loss of generality. While the MPM/Siacci RF is single/vertical plane only, swerve jump components can be normal to this plane. This is sensed for the TIRF y-right definition for consistency.

The following definitions for 6 degrees of freedom (6DOF) models are included for the modeler's general understanding and appreciation of the approximations involved, although it will be made apparent that they are not used directly in aerial gunnery modeling. It is noted that the ARL author refers to "approximate equations of motion" for 6DOF models which is true, but without explaining that they are the most accurate.

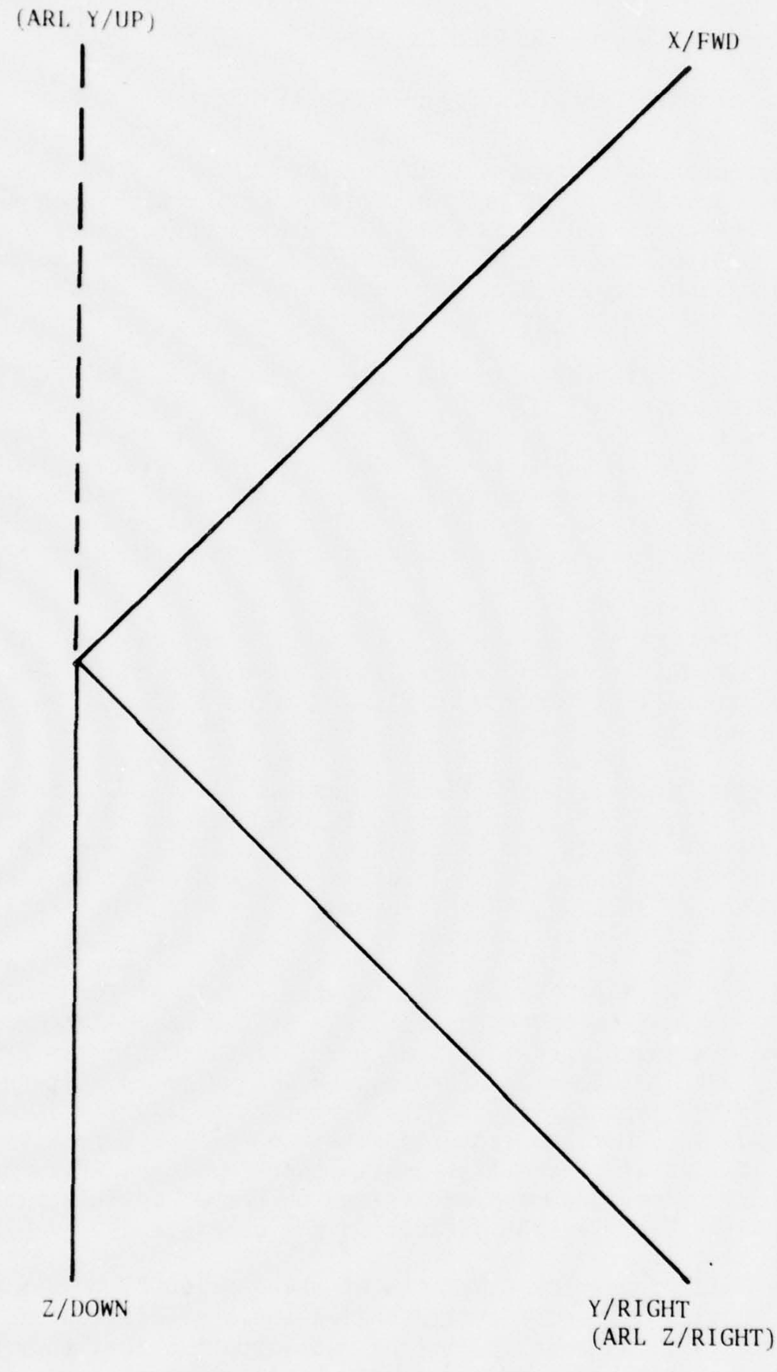

Figure 4. Basic Versus ARL Reference Frame

Flying out the trajectory, the 6DOF model requires a trajectory flight RF (TFRF), denoted 1, J, K by ARL, analogous to the wind/stability (axes) RF (WRF/SRF) of Section V. It is indexed with respect to the TIRF and updates/defines relative bullet cg translations and rotations.

The bullet body (BB) RF, defined A, B, 3 by ARL, is essentially identical to the TFRF excepting BB roll rotation after the yaw on the bullet damps out. Thus, the effect of yaw is the essential differentiation of the 6DOF from simplified models. Note, the BBRF Y/ARL-3 and Z/ARL-B rotate at the BB spin rate about X/ARL-A. The BBRF is initialized to the TFRF/TIRF by the rotations, θ-sub Y and ψ-sub Y which are defined in the next section. A yawed BBRF X/ARL-A precesses/nutates about the TFRF X/ARL-1. To account for its orientation, ARL defines a 1, 2, 3 RF, a bullet nutation-accounting RF (BNRF), where 1 is velocity/flight vector directed; i.e., 1 is the same in the 1, 2, 3 and 1, J, K RFs. Then nutation of 2 from J (or 3 from K) defines the precession angle, ARL ϕ, while the magnitude of the yaw/sigma angle, ARL δ, is defined from 1 to A in the 1, 2 plane.

It is noted that the MPM/CSS requires only the definition of the TIRF in the ATAGAM FIRF. The 6DOF model also requires the TFRF, BBRF, and BNRF.

SECTION VII

TRANSFORMATIONS

Sections 4 and 5 of Reference 2 present a clear, concise treatment of the standard transformations. However, methods of uncoupling the various parts may not be readily evident to the unfamiliar. For example, the P of subsection 5.1 could be related to projectile/bullet position, the O to the body cg, and the OI to the origin of the FIRF. While mathematically correct, the development would readily become mind boggling in definition of the X, Y, and Z derivatives of P with respect to O.

In practice, each aircraft and bullet cg is initiated and updated in the FIRF, because the equations of motion are both independent and definable in this RF. For each axis, both translational and rotational motions are updated by the Taylor Series equation:

$$f(t) = f(o) + \dot{f} \cdot t + \ddot{f} \cdot t^2/2! + \dddot{f} \cdot t^3/3! + \text{HOTs} \qquad (1)$$

where f = $x/y/z$ translation and $\phi/\theta/\psi$ rotation

t = time from initiation, zero time

HOTs = higher order terms (usually neglected).

As previously noted, the bullet equations of motion (excluding jump) are usually defined in a single plane and then transformed to the FIRF.

It is noted that the time increment in iterative models is kept small so that the HOTs have negligible effect. Many ignore the third-derivative/jerk term. Accuracy is affected by both iteration interval and truncations of the series. The resultant angles are Euler attitudes of the body RF with respect to the MIRF indices.

In the real world, the Euler attitudes are sensed by pickoffs on the gyro gimbals. If inertial, the platform contains rate gyros and (translational) accelerometers. However, the rate gyro and accelerometer outputs are used internally. That is, the normal outputs of the platform are attitudes and translations. Thus, models that use real world inputs do not require accountings of time and derivatives to determine Euler attitudes. However, instrumentation is most often found lacking and occasions arise where angular derivatives in the body RF must be obtained. Of course, differentiation is required with the well known noise problems. Most models define body rates and Euler attitudes must be obtained from them. Thus, the required rotational transformations (RTs) can be expressed as:

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 & 0 & -S\theta \\ 0 & C\phi & S\phi C\theta \\ 0 & -S\phi & C\phi C\theta \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \tag{2}$$

and

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & S\phi T\theta & C\phi T\theta \\ 0 & C\phi & -S\phi \\ 0 & S\phi/C\theta & C\phi/C\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{3}$$

where S, C, T prefixes are shortening abbreviations for sine, cosine, and tangent functions. Due to sequential rotations in their definition (Reference 2), these RTs are not orthogonal. Properties of orthogonality are defined later in this section.

Before general use of large digital processing machines, limited/analog processors required linearizing p/q/r to phi-dot/theta-dot/psi-dot for specified initial conditions; i.e., near zero psi, theta, and phi where sine/cosine functions are approximately 0/1. The restrictions on linearizations are obvious.

Because the Euler-sensing gyro/platform is affixed to the body, occasions arise quite frequently where understanding that Euler and body rates are not identical is difficult for some. Note that the MIRF axes remain inertially fixed and sense any of the arbitrarily oriented body motions in their respective axes.

Having established the relationships between Euler and body axes rates and the definition of the Euler attitudes, the standard RTs between RFs can be expressed as:

$$RX(A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & CA & SA \\ 0 & -SA & CA \end{bmatrix} \tag{4}$$

$$RY(B) = \begin{bmatrix} CB & 0 & -SB \\ 0 & 1 & 0 \\ SB & 0 & CB \end{bmatrix} \tag{5}$$

$$RZ(C) = \begin{bmatrix} CC & SC & 0 \\ -SC & CC & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (6)$$

where S&C prefixes are shortening abbreviations for sine and cosine functions, RX/RY/RZ is the RT about the X/Y/Z axis, A/B/C is the associated RH rotation angle from the reference index. It is noted that the development allows independent determination of components in each rotation plane (i.e., the XY plane for Z-axis rotations, etc.) separately, while the results for all of the ordered rotations are given by the ordered product of their RTs.

The majority of the objects of interest requiring transformation are associated with the attacker airframe whose states must be updated to define flight, while the only object of interest in the WRF is the velocity vector. Thus, while object-to-body-to-wind-to-earth and reverse transformations can be defined correctly, they contain needless complications. Relating objects to the body and using body-to-earth and reverse transformations minimizes the number of transformations required. Accordingly, the standard transformations are presented as follows.

EARTH-TO-BODY AND REVERSE

$$PQB = [RX(\phi) \cdot RY(\theta) \cdot RZ(\psi)] \cdot PQE = RBE \cdot PQE \qquad (7)$$

where PQB/PQE is the body/earth column matrix of X, Y, and Z position coordinates/components of some point of interest (Q) such as the target, projectile, gun muzzle, sight, etc. The psi, theta, phi ordering in matrix algebra is evidenced in the pre-multiplications.

P/V/A first-letter notation identifies position/velocity/acceleration (vectors). The intermediate letter(s) identify the point/object of interest, while the last letter identifies the RF. Coordinates/components are identifiable by the X, Y, or Z ending added to the vector notations. Elements of the RTs are conventionally identified by first letter C, second letter symbol for object, third letter notation for output RF axis component, and fourth letter for input RF axis contributing component.

Using the orthogonality principle of RTs,

$$RBA^T = RBA^{-1} = RAB \qquad (8)$$

where A is one RF, B is another, and AB implies transforming from B to A coordinates,

$$PQE = RBE^T \cdot PQB = REB \cdot PQB \qquad (9)$$

The AB notation used above is totally reversed or used inconsistently by some modelers. Consistency with that notation has been used in the following discussions. It is also noted that

$$R1(D)^T = R1(-D) \text{ where } D = \text{associated angle for any/1 axis} \tag{10}$$

and

$$RBE^T = RZ(-\psi) \cdot RY(-\theta) \cdot RZ(-\phi) \tag{11}$$

i.e., rotating backwards through the angular transformations is the same as the transpose.

Standard notation, proffered by and received from AFAL/RWT-2 in coordination, is repeated verbatim as Appendix A of this report. Note reverse AB implication. Coordinated agreement/refinement and a listing of common variables is considered to be a good basis for a companion report.

WIND-TO-BODY AND REVERSE

Flight definitions define the wind vector and angular components alpha and beta from the wind to either the body or other FBRF. Usually, the angles between the WCRF/other FBRF frame and the body are available so rather than using multiple transformations, the angles to the body can be obtained by a simple, directed-sum. Thus, the basic wind/body transformations are:

$$PQB = [RY(\alpha) \cdot RZ(-\beta)] \cdot PQW = RBW \cdot PQW \tag{12}$$

and

$$PQW = RBW^T \cdot PQB = RWB \cdot PQB \tag{13}$$

where PQW is defined similarly to PQB/PQE/ for X, Y, Z components in the wind axes. Note that accounting for left-hand sensed beta is accomplished by inputting negative beta to the RHRT. Note also the absence/identity of the roll (x-axis) RT.

EXTERIOR BALLISTICS

There are several factors that are required to initialize the fired bullet; i.e., the launch velocity, its position and orientation, and the wind angle on the projectile if windage jump is considered.

The essential launch velocity vector (VL) transforms are wind-to-body aircraft velocity vector (VA), addition of muzzle velocity vector (VM) and muzzle tangential velocity vector (VMT) in the same frame, and rotating

their sum into the FIRF. VA in the BRF can be expressed as:

$$VAB = RBW \cdot VAW \tag{14}$$

where VAB/VAW is a column vector of the X, Y, and Z velocities in the BRF/WRF. The components in the WRF are VA, 0 and 0; so that RBW•VAW can be reduced to the first column times VA (scalar) if desired for computational efficiency.

The muzzle velocity vector in the BRF can be expressed as:

$$VMB = [RY(EG) \cdot RZ(AG)]^T \cdot VMG = RBG \cdot VMG \tag{15}$$

where EG/AG is the theta/psi rotations of the gun from the BRF. Again, VMG components in the gun RF are VM, 0, 0; so RBG•VMG can be reduced if desired.

The gun muzzle and sight/LCG are usually located at significant distances from the aircraft cg. Aerodynamic flexures are generally unknown quantities so that these elements are considered fixed in the airframe. Since a point on a rigid arm rotates with the center/cg, the sight/LCG offsets do not affect their functionings. However, the tangential velocity of the muzzle adds vectorially to the launch velocity. The tangent velocity vector in the BRF can be expressed as:

$$VMTB = \omega \otimes R = \tilde{\omega} \cdot PMB \tag{16}$$

where PMB is the muzzle position vector in the BRF, and

$$\tilde{\omega} = \begin{bmatrix} o & -r & q \\ r & o & -p \\ -q & p & o \end{bmatrix}.$$

The tilde operator allows cross-product operations to be defined by matrix multiplication. Thus, the components are:

$$VMTBX = qz - ry \tag{17}$$

$$VMTBY = rx - pz \tag{18}$$

$$VMTBZ = py - qx \tag{19}$$

To ballpark the effect of tangential velocities on highly maneuverable aircraft, the upper limits of p/q/r are about 4/0.35/0.1 rad/sec while typical muzzle displacements are 10-20/0-10/0-5 feet. Accordingly, comparisons of tangential velocities of a few ft/sec to average bullet velocities of a few thousands ft/sec indicate an effect of about a thousandth of a rad/sec change in direction/time-of-flight (TOF). Because of their small effect, some models ignore tangential velocities.

The total launch velocity in the BRF and FIRF can be expressed as:

$$VLB = VAB + VMB + VMTB \tag{20}$$

and

$$VLI = REB \cdot VLB \tag{21}$$

It is noted that the vectorial summing in these equations inherently accounts for what ballisticians have called aerodynamic jump; i.e., the vector sum of aircraft and muzzle velocity vectors.

The projectile position at launch in the FIRF can be expressed as:

$$PPLI = PAI + REB \cdot PMB \tag{22}$$

where PAI is the inertial firer-cg location vector, defined by Equation (1).

The wind/yaw angle on the projectile is simply the directional difference of the launch velocity vector from the muzzle velocity vector. The muzzle velocity vector in the FIRF is simply:

$$VMI = REB \cdot VMB \tag{23}$$

Thus, the launch and muzzle velocity vector directions can be expressed as:

$$\psi\text{-sub } L = \arctan (VLIY/VLIX), \tag{24}$$

$$\psi\text{-sub } M = \arctan (VMIY/VMIX), \tag{25}$$

$$\theta\text{-sub } L = \arcsin (-VLIZ/abs(VLI)), \tag{26}$$

and

$$\theta\text{-sub } M = \arcsin (-VMIZ/abs(VMI)), \tag{27}$$

where abs (A) = magnitude of A, i.e., the root-sum-square of the components. Accordingly, the bullet yaw components can be expressed as:

$$\psi\text{-sub } Y = (\psi\text{-sub } M) - (\psi\text{-sub } L) \tag{28}$$

and

$$\theta\text{-sub } Y = (\theta\text{-sub } M) - (\theta\text{-sub } L) \tag{29}$$

The spatial bullet yaw (Σ) can be defined by either:

$$abs(VLI)abs(VMI) \cos (\Sigma) = VLI \odot VMI \tag{30}$$

or

$$\Sigma = \text{arc cos } (\cos(\psi\text{-sub Y}) \cos (\theta\text{-sub Y})) \tag{31}$$

The high dynamics of precessional/nutational/corkscrewing motion of the yawed projectile requires an extremely small iteration interval, even though the ballistics flight equations are very simple in comparison to aircraft. Thus, even CDC 6600 processor time is about 100 times real-time. Because the corkscrew essentially damps out in the first 500 feet of the aerial trajectories of interest and its disturbance effects are predictable from analysis of 6DOF outputs, use of such modeling in general analyses would be inefficient.

The effects of yaw are obviously direct lift, magnus lift, and increased drag. ARL has shown in References 3 and 4 that increased drag can be mechanized by the addition of an exponential term in the velocity equations. Conversion to basic RFs requires consideration of different indexing including the delta altitude term. Thus, the remainder of yaw effects definition involves windage/swerve jump.

Figure 5 shows the geometry of windage/swerve jump with the TFRF X/velocity-vector into the page. The yaw/sigma (Σ) angle from the flight vector to the projectile nose direction is represented by the radius of the circle. The resultant effects over the corkscrew path of the projectile can be expressed as a radially outward effect (vector a) and a tangential effect (vector b) with regard to the projectile's initial orientation. If the radial and tangential effects were directly correlatable with the initial orientation, then, the radially-outward/tangential effect could be attributed to direct/magnus lift. The drawing of the vectors on top of the circle are included for ease in visualizing the cause and effect, while their orientation on the lower right of the circle are included to illustrate arbitrary orientation of initial conditions, defined by ARL ϕ. The magnitudes of a and b are derived as a function of P (which is closely correlatable to projectile range) for specified initial orientations. Thus, even if there is a phasing between initial orientation and effect, the values of a and b will reflect the effect. Thus, it follows directly that:

$$\text{delta-y} = P \cdot \Sigma \, (a \sin \phi + b \cos \phi) \tag{32}$$

and

$$\text{delta-z} = -P \cdot \Sigma \, (a \cos \phi - b \sin \phi) \tag{33}$$

where P = Siacci space function.

It is noted that:

$$\Sigma \cos \phi = \theta\text{-sub Y} \tag{34}$$

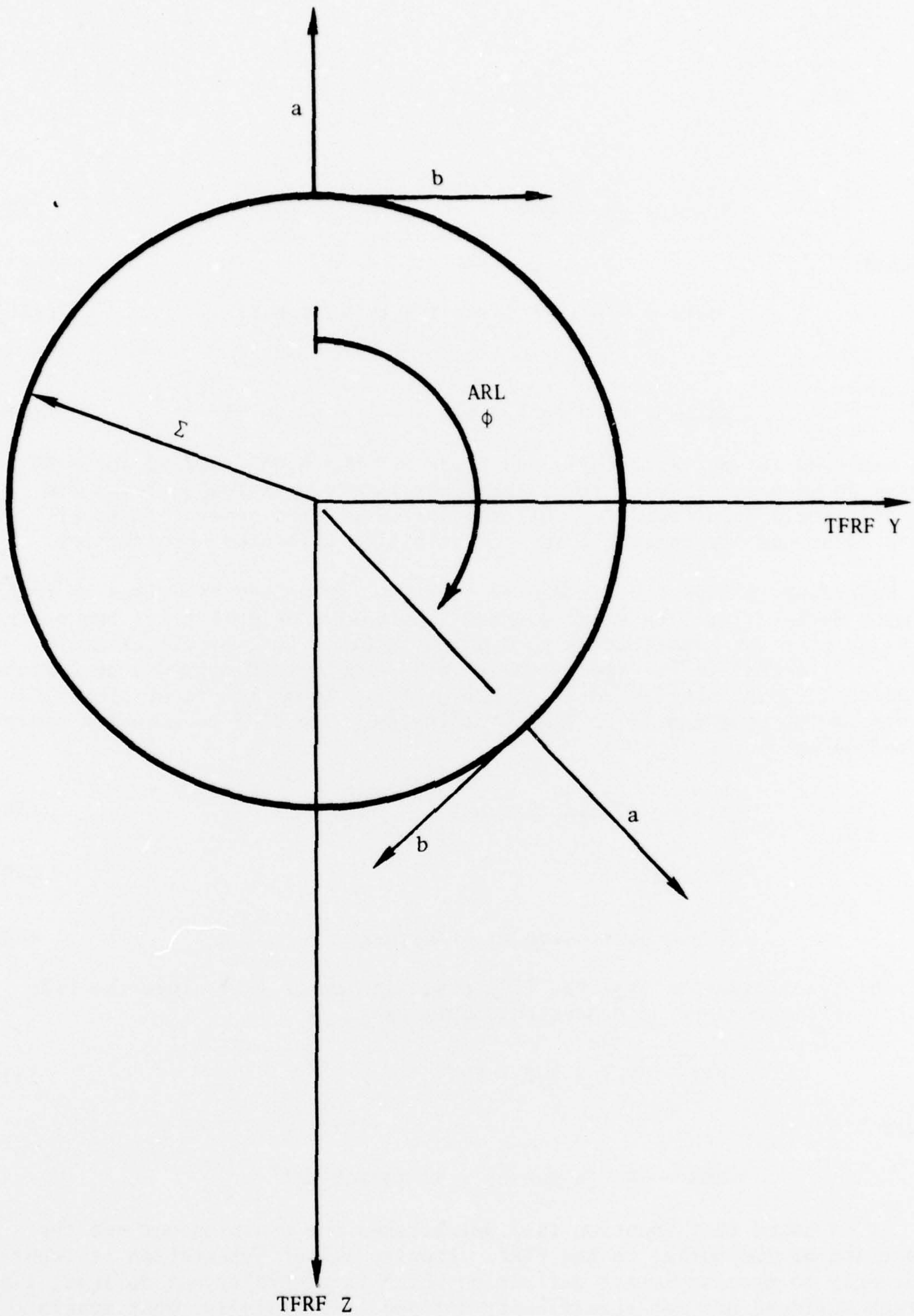

Figure 5. Direct and Magnus Lift Effects

and

$$\Sigma \sin \phi = \psi\text{-sub } Y \tag{35}$$

so that

$$\text{delta-}y = P\ (a \cdot \psi\text{-sub } Y + b \cdot \theta\text{-sub } Y) \tag{36}$$

and

$$\text{delta-}z = -P\ (a \cdot \theta\text{-sub } Y - b \cdot \psi\text{-sub } Y) \tag{37}$$

The modified ARL estimates of a and b are 0.0 and 0.037 rad/rad for M-50 series 20 mm projectiles. Thus, the terms involving a in Equations (36) and (37) could be dropped for this special case. The general forms of these equations are desirable for compatibility with other projectiles.

Referring to Figure 6, it can be seen that an ambiguity arises in transforming delta-Z into the P and Q axes. Obviously, delta-Z times the cosine of θ-sub L is the contribution to Q which is GVD. But, by definition delta-Z is normal to P. Specifically, at θ-sub L of 90 degrees, no component of delta-Z can be attributed to either Q or P. Thus, the first step is to transform P-Q coordinates to TIRF coordinates. The TIRF coordinates can be expressed as:

$$X = P - Q \sin\ (\theta\text{-sub } L) \tag{38}$$

$$Y = \text{delta-}Y \tag{39}$$

$$Z = Q \cos\ (\theta\text{-sub } L) + \text{delta-}Z \tag{40}$$

The transformation from the TIRF position vector (PPT) into the FIRF vector (PPI) can then be expressed simply as:

$$PPI = PPLI + REB \cdot PPT \tag{41}$$

where

$$REB = (RY\ (\theta\text{-sub } L) \cdot RZ\ (\psi\text{-sub } L))^T.$$

It is noted that Equation (41) establishes the position but not the direction of the bullet in the FIRF. Precise bullet orientation is important only to precise impact definition which is generally not defined; i.e., target surfaces are not specifically defined. Accordingly, most models ignore the gravity rotation which is generally less than a degree and windage jump components which are generally about a few milliradians. Derivation of bullet direction is relatively simple. The TFRF/TIRF Y is always in a level FIRF plane so that

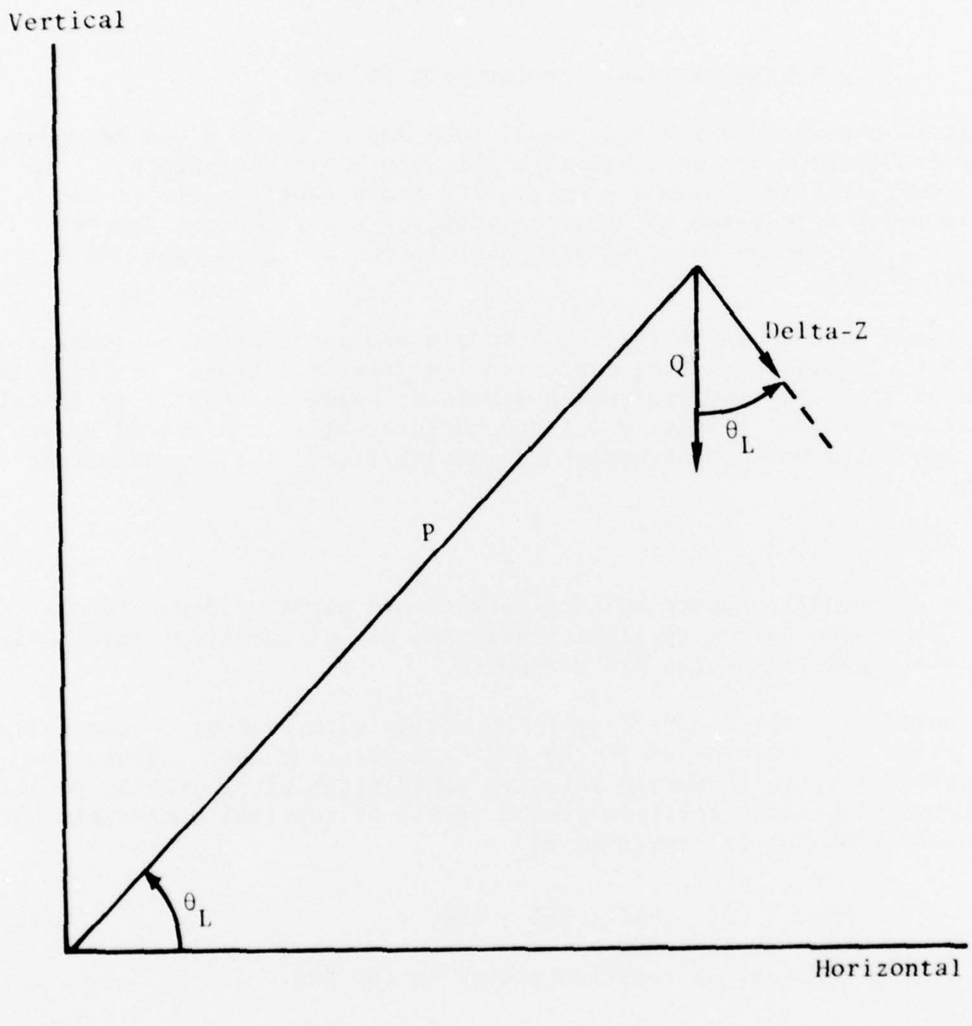

Figure 6. Delta-Z Ambiguity in P-Q

$$\omega\text{-sub Y} = \text{-V-sub B}/(\text{g} \cdot \cos(\theta\text{-sub B}) \qquad (42)$$

where ω-sub Y is the gravitational rotation rate

V-sub B = bullet velocity in ft/sec

g = gravitational acceleration ft/sec^2

Starting at launch where θ-sub B is equal to θ-sub L, θ-sub B can be updated by the Taylor Series equation. Finally, the jump angle components, θ-sub J = -delta-Z/P and ψ-sub J = delta-Y/P, are directly additive to θ-sub B and ψ-sub B (= ψ-sub L), respectively, as a first order approximation, i.e., the change in gravitational rotation due to θ-sub J is a second-order effect.

It has been made apparent that 6DOF models are inefficient in general ATAGAM, due to excessive running time. In iterative solutions, accuracy is a function of iteration interval which should be apparent from basic calculus. For general aerodynamic flight, a 0.1-second interval is considered to be the best compromise between accuracy and running time. The approximation is evident.

BULLET-TO-SIGHT

Quite a few models compare bullet, target, and pipper coordinates in the FSRF. Since the target-to-sight transforms are of identical form, only the bullet-to-sight transforms are presented.

It is noted that the aircraft cg is in motion with respect to the bullet launch position and coordinates in the FSRF are relative ones. Thus, the first transformation is to derive relative coordinates of projectile position. The bullet-position-sight-relative vector (PPSI) of inertial components with respect to the FSRF can be expressed as:

$$\text{PPSI} = \text{PPI} - \text{PAI} + \text{REB} \cdot \text{PSB} \qquad (43)$$

where PSB is the FSRF-origin location vector in the BRF.

The transformation to the BRF can be expressed as:

$$\text{PPSB} = \text{RBE} \cdot \text{PPSI} \; ; \qquad (44)$$

and, finally, the transformation to the FSRF can be expressed as:

$$\text{PPS} = \text{RSB} \cdot \text{PPSB} \qquad (45)$$

where RSB is RY (θ-sub FS); θ-sub FS being the rotation of the FSRF from the BRF. Of course, abs (PPS) is the relative projectile range, and the angular positions with respect to the FSRF origin in the FSRF can be expressed as:

$$\psi\text{-sub P} = \arcsin\ (PPSY/abs(PPS)) \tag{46}$$

$$\theta\text{-sub P} = \arcsin\ (-PPSZ/abs(PPS)) \tag{47}$$

Presentation of all projectile coordinates at a specific instant/iteration is analogous to the visibly-sensible tracer line. Sequential presentation of their intersection (requiring interpolation of fly-by) in the impact plane is analogous to what would be seen if the projectiles fuzed, regardless of hit, at fly-by. The latter is more important in modeling hit probabilities.

The target, projectile(s) (at impact-plane intersection), and the pipper positions in the sighting RF allow calculations of accuracies and statistics.

ANGLE-OFF-TRANSFORMATIONS

In modeling, angle-off (AO) is defined to be the angle between the LOS range vector (LOSR) and the target velocity vector. In the past, AO has been defined as an acute angle, due to geometric limitations on accurate gunsight operations. The following derivations have been generalized.

The position of the attacker's sight (PSI) in the FIRF is simply the last two terms of Equation (43);

$$PSI = PAI + REB \cdot PSB \tag{48}$$

Then, the FIRF LOSR (PLI) can be expressed as

$$PLI = PTI - PSI \tag{49}$$

where PTI is the target-cg location in the FIRF. It is noted that angle-off is defined for the conical angle of PLI about the target velocity vector; i.e., any AO that is defined by a point on the cone is not distinguished from another. Thus, AO can be defined by dot producting PLI with the target velocity vector (VTI) in the FIRF; i.e.,

$$\cos\ (AO) = PLI \odot VTI/(ABS(PLI) \cdot abs(VTI)) \tag{50}$$

It is also noted that angle-off is usually considered a cursory measure and most models neglect the PSB term.

Another term, angle-off-the-tail (AOT) is used as a gross measure of target aspect. AOT is sometimes defined as the negative of PLI dot-producted with a unit vector (UV) along the target negative X axis which, of course, is identical to the dot product of the position vectors; i.e.,

$$UVTXI = REB \cdot (1,0,0)^T \tag{51}$$

where UVTXI is the FIRF transform of the TBRF (target BRF) X UV.

1 is the X component on the UV along the TBRF X

and,

$$\cos\ (AOT) = (PLI \odot UVTXI)/abs(PLI) \tag{52}$$

Note that REB is defined for the TBRF in this equation.

In some models, target aspects/views are defined, using the basic psi, theta, phi conventions. This requires definition of the negative (PNLI) of PLI in the TBRF and then defining the directions of the PNLI in the TBRF. This can be expressed as:

$$PNLB = RBE \cdot PNLI \tag{53}$$

where PNLB is the PNLI vector in the TBRF. Again, RBE is defined for the TBRF.

Then, the target-aspect heading (HASP), pitch (PASP), and roll (RASP) angles can be expressed as:

$$HASP = \arctan\ (PNLBY/PNLBX) \tag{54}$$

$$PASP = FCN1\ (HASP,\ PARG) \tag{55}$$

$$PARG = \arcsin\ (PNLBZ/abs(PNLB)) \tag{56}$$

$$RASP = \arctan\ (PNLBZ/PNLBY) \tag{57}$$

where

PARG is the pitch aspect angle argument

FCN1 = PARG for HASP in forward hemisphere
180-PARG for HASP in rear hemisphere.

The ambiguities for HASP/RASP at $\pm\ \pi/2$ can be handled in any consistent manner, being usually assigned to the positive semi-circle. It is noted that aspects in the vertical plane of the target define positive 90-degree roll aspects which should not be confused with target attitude in the FIRF.

Also, note that rotation of pilot/firer up about the LOSR does not change these target aspects. That is, whether the firer's head is upright, tilted, or inverted in the FIRF does not change the fact that he views the target's top/bottom, left/right side, and front/rear views/aspects from the specified aspect angles.

Finally, crossing angle is a term that is used to define the bullet aspect with respect to the target fly-by. It is identically derived to AOT and/or aspects with the exception of using the negative of the bullet velocity vector instead of PNLI. Of course, striking angles against and unit projected areas of front, right side, and bottom views are defined by dot-producting the negative of the bullet velocity vector (VNB) with UVs along the TBRF X, Y, and Z axes, respectively. Note that negative products of the UV and the VNB component in its direction indicate rear, left-side, and top views. The total projected area of a view is simply the normal area of the view times its unit projected area. Where vulnerabilities of the target are given by azimuth and elevation aspects with elevation as the combination of roll and pitch aspects, the complement of the striking angle against the top/bottom view can be used to define this elevation.

MAJOR CONFUSION FACTORS IN TRANSFORMATION

Confusion rarely occurs when it is made clear that coordinates of a specific object are being transformed from one coordinate system to another. In many cases, the model develops multiple transformations without reference to what is to be transformed and to which RFs. Also, some models will refer to Euler attitudes as Earth angles rather than body angles to Earth indices/references. Specific comments of what is being transformed and the RTs involved will avoid such confusion.

SECTION VIII

RECOMMENDATIONS AND CONCLUSIONS

The preceding discussions are directed to provide a common basis for development of ATAGAM. Compliance with these preferred standards should simplify their structure and general comprehension.

The discussions understandably fall short of total comprehension. A first addendum may be envisioned to expound on the various models used to define flight. This is the major difference between most tactical models. The simpler models consider flight control only to rotate the flight vector, neglecting direct lift effects along with cross-coupling, phugoid, and short period dynamic modes of the airframe. Another could address the paper pilot, the most controversial aspect of all-digital codes. Besides neuro-muscular lags and decision-delay functions, high integral gain along with optimized proportional and derivative gains is sometimes included in the definition of a second-order paper pilot to mislead the unfamiliar into believing that the flight model encompasses all dynamics of the airframe. Others could be directed to explain and compare various interpolation, integration, differentiation, smoothing, atmosphere, matrix algebra routines, etc. that make up the standard subroutines of the model.

It is intended that this treatise plus other/addenda will provide a basis for modular construction of aerial gunnery models whose capabilities and limitations are readily identifiable.

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APPENDIX A

RACEE Simulation-FORTRAN Kinematic Variables Mnemonics

The FORTRAN kinematic variables have been named by the conventions below. It is hoped that after reading this appendix, their interpretation will be both uncomplicated and precise. The intention of the naming convention is to distinguish between five properties of these variables. These properties are as follows:

1. Order of derivative and type of variable. For example, position, velocity, acceleration, angular velocity, transformation matrix, etc.

2. Physical object or mathematical object concerned. For example, attacker aircraft, target, inertial frame, air mass frame, line of sight frame, etc.

3. Another object (physical or mathematical) which the variable of the first object is in relation to. For example, velocity (variable) of target (first object) with respect to attacker (second object). Generally, the same set of objects could be used here as in Number 2.

4. The coordinate frame into which components of the vectors are resolved. Noting that most of the variables in Number 1 are vectors, and that to operate numerically on a vector, it must be resolved into a coordinate frame, these frames must be specified. Hence, for a complete description of the variable given in the example of Number 3, it would be necessary to specify that the vector was resolved into attacker body axes, inertial axes, or line of sight axes, etc.

5. It is recognized that there are other attributes that might be sytematized, such as noise-corrupted versus perfect variables. This could be recognized as a fifth property.

The mnemonics are determined by the following conventions:

FORTRAN name = "uvwxyz"

where "uv"	=	PS	position
		VL	velocity
		AC	acceleration
		SF	specific force = acceleration - gravity
		OM	omega (angular rate)

OD	omega dot (angular acceleration)
SI	ψ (general Euler azimuth angle about z-axis)
TH	θ (general Euler elevation angle about y-axis)
PH	ϕ (general Euler roll angle about x-axis)
T	transformation matrix,

where

w or x or y =	A	attacker body
	T	target body
	I	inertial frame
	W	wind frame of attacker
	L	line of sight frame to target (non-roll stabilized)
	M	missile body
	B	bullet
	V	bullet velocity frame as in "MISS"
	S	scoring frame as in "MISS",

and where

z =	(blank)	uncorrupted (exact) program variable
	M	measured varible, e.g., true variable corrupted by noise for input to airborne algorithm

Some examples of this convention are:

PSTAI	target distance north, east, and down from attacker
VLAIA	attacker inertial velocity in body axes
VLAWA	attacker air mass velocity
OMAIA	attacker angular rate, i.e., p, q, r
SITAL	target azimuth angle from attacker
TIA	transformation from inertial to attacker body axes
SFTIT	target specific force vector (i.e., SFTIT(3) = -target load factor • 32.174)
SIAIA	attacker heading angle (of body)

One of the benefits of this convention is rapid checkout of vector operations. For example, in any addition or subtraction of vectors, the fifth letter

must be the same. In any transformation of vectors, the fifth letter of the transformed variable and the fifth letter of the original vector determine which transformation to use. An example of this would be

PSTAA = (TIA) (PSTAI)

In addition or subtraction, the third and fourth letters must be consistent with the operation. For example,

VLTAI = VLTII - VLAII

or

ACTII = ACTAI + ACAII

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AFATL/DLYV	1
AFATL/DLYD	10
ASD/SDX	1
AFA/DFASC	1
TAC/INAT	1

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